## Exercise 27

For the following exercises, consider the function $f(x) = \sqrt{1 - x^2}$. (*Hint*: This is the upper half of a circle of radius 1 positioned at $(0, 0)$.)

Use the preceding exercise to find the aproximate area between the x-axis and the graph of f over the interval $[-1, 1]$ using rectangles. For the rectangles, use squares 0.4 by 0.4 units, and approximate both above and below the lines. Use geometry to find the exact answer.

Solution

The area below the graph of $f(x) = \sqrt{1 - x^2}$ can be found using rectangles in two ways. In the first way the top left of each rectangle touches the graph, and in the second way the top right of each rectangle touches the graph.
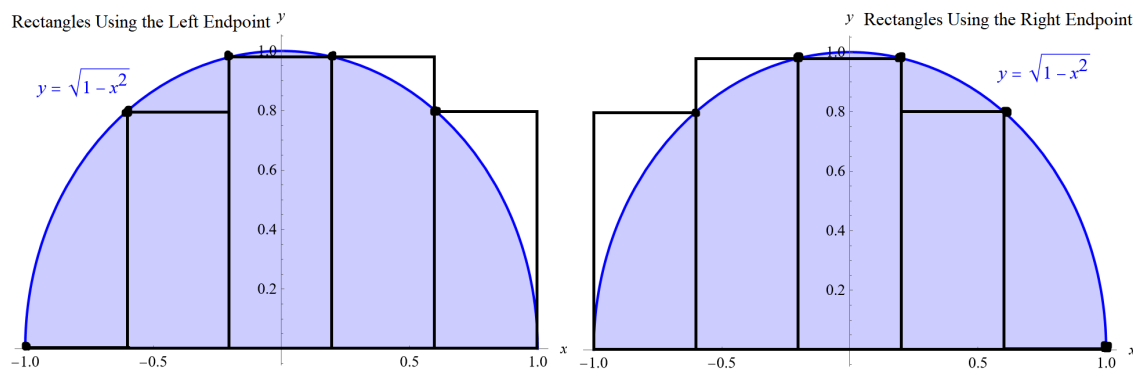
Add the areas of all the rectangles (each with width 0.4) in each case.

$$A_{\text{Left}} \approx (0.4)f(-1) + (0.4)f(-0.6) + (0.4)f(-0.2) + (0.4)f(0.2) + (0.4)f(0.6)$$

$$\approx (0.4)\sqrt{1 - (-1)^2} + (0.4)\sqrt{1 - (-0.6)^2} + (0.4)\sqrt{1 - (-0.2)^2} + (0.4)\sqrt{1 - (0.2)^2} + (0.4)\sqrt{1 - (0.6)^2}$$

$$\approx (0.4)(0) + (0.4)(0.8) + (0.4)(0.98) + (0.4)(0.98) + (0.4)(0.8)$$

$$\approx 1.424$$

$$A_{\text{Right}} \approx (0.4)f(-0.6) + (0.4)f(-0.2) + (0.4)f(0.2) + (0.4)f(0.6) + (0.4)f(1)$$

$$\approx (0.4)\sqrt{1 - (-0.6)^2} + (0.4)\sqrt{1 - (-0.2)^2} + (0.4)\sqrt{1 - (0.2)^2} + (0.4)\sqrt{1 - (0.6)^2} + (0.4)\sqrt{1 - (1)^2}$$

$$\approx (0.4)(0.8) + (0.4)(0.98) + (0.4)(0.98) + (0.4)(0.8) + (0.4)(0)$$

$$\approx 1.424$$

These are only approximations. The exact area under the curve is half the area of a circle with radius 1.

$$A = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2} \approx 1.571$$